Six Basic Function Transformations

	Algebraic Transformation In Words	Algebraic Transformation In Symbols	Describe it As...	Effect on Points
Horizontal Transformations	Replace x with $(x + c)$	$f(x + c)$	"Horizontal translation $-c$ units"	Subtracts c from all x-values
	$f(x - 3)$ translates the graph to the right 3 units, $f(x + 3)$ translates the graph to the left 3 units			
	Replace x with cx	$f(cx)$	"Horizontal dilation by a factor of $\frac{1}{c}$"	Divides all x-values by c
	$f(3x)$ is narrower than $f(x)$, $f\left(\frac{x}{3}\right)$ is wider than $f(x)$			
	Replace x with $-x$	$f(-x)$	"Reflection over the y-axis"	Multiplies all x-values by -1
Vertical Transformations	Add $+c$ to the end of the function	$f(x) + c$	"Vertical translation c units"	Adds c to all y-values
	$f(x) + 3$ translates the graph up 3 units, $f(x) - 3$ translates the graph down 3 units			
	Multiply the entire function by c	$cf(x)$	"Vertical dilation by a factor of c"	Multiply all y-values by c
	$3f(x)$ is taller than $f(x)$, $\frac{1}{3}f(x)$ is shorter than $f(x)$			
	Negate the entire function	$-f(x)$	"Reflection over the x-axis"	Multiplies all y-values by -1

Some Patterns to Help Make Sense of This

- Horizontal transformations all involve replacing x
  - Replacing x with $-x$ does a reflection over the y-axis, because we are flipping the graph side to side. A horizontal flip.
- Vertical transformations all involve doing something to the entire function
  - Negating the entire function does a reflection over the x-axis, because we are flipping the graph top to bottom. A vertical flip.
- Translations all involve adding/subtracting
- Dilations all involve multiplying/dividing
- Horizontal moves feel "backwards".
  Replacing x with $(x+5)$ does not add 5 to x-values, it subtracts 5 from x-values. Replacing x with $2x$ does not multiple x-values by 2, it multiplies x-values by $\frac{1}{2}$. These feel "backwards".
- Vertical moves are not backwards.
  $f(x)+5$ really does add 5 to y-values. $2f(x)$ really does multiply y-values by 2.